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3 (Sem-5/CBCS) STA HC 1

2021

(Held in 2022)

STATISTICS

(Honours)

Paper : STA-HC-5016

(Stochastic Processes and Queuing Theory)

Full Marks : 60

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

1. Answer the following as directed:

1×7=7

(a) The mean of X in terms of the probability generating function (p.g.f.) of X is given by

(i) $P''(1)$

(ii) $P'(1)$

(iii) $P'(S)$

(iv) None of the above

(Choose the correct option)

Contd.

(b) Define stochastic processes.

(c) An irreducible Markov chain contains

- (i) one closed set
- (ii) two closed sets
- (iii) three closed sets
- (iv) All of the above

(Choose the correct option)

(d) Difference of two independent Poisson processes is also a Poisson process.

(State True or False)

(e) State one property of transition probability matrix.

(f) The average queue size in M/M/1/1 queueing model is

- (i) 1
- (ii) 0
- (iii) 2

(iv) None of the above

(Choose the correct option)

(g) The interval between two successive occurrences of a Poisson process has an _____ distribution.

(Fill in the blank)

2. Answer the following questions briefly :

2×4=8

(a) Define bivariate probability generating function of a pair of random variables X and Y.

(b) Define Markov chain with an example.

(c) State any two postulates for Poisson process.

(d) Define traffic intensity. State the condition for existence of steady state of M/M/1 queueing model with infinite system capacity. $1+1=2$

3. Answer **any three** of the following questions :

5×3=15

(a) Let X be a Poisson variate with p.m.f

$$p_k = P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}; k = 0, 1, 2, \dots$$

Find the mean and variance of X using probability generating function (p.g.f) of X.

(b) Write an explanatory note on specification of stochastic processes.

(c) Derive the probability distribution of number of customers in $M/M/1$ queuing model with finite system capacity.

(d) Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is $\frac{1}{3}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$. Given that May 1 is a dry day, find the probability that May 3 is also a dry day.

(e) Suppose that customers arrive at a service counter in accordance with a Poisson process with mean rate of 2 per minute ($\lambda = 2/\text{minute}$). Then the interval between any two successive arrivals follow exponential distribution with mean $\frac{1}{\lambda} = \frac{1}{2}$ minute. Find the probability that the interval between two successive arrivals is —

- (i) more than 1 minute;
- (ii) 4 minutes or less;
- (iii) between 1 and 2 minutes.

4. Answer **either** (a) **or** (b) :

(a) (i) Define matrix of transition probabilities. Let $\{X_t, t \geq 0\}$ be a Markov chain with three states 0, 1 and 2 with transition matrix

$$\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

and the initial distribution

$$P\{X_0 = i\} = \frac{1}{3}, \quad i = 0, 1, 2$$

Find $P\{X_1 = 1/X_0 = 2\}$;

$P\{X_2 = 2, X_1 = 1/X_0 = 2\}$ and

$P\{X_2 = 2, X_1 = 1, X_0 = 2\}$.

$$1+1+2+2=6$$

(ii) Write a short note on Markov chain as graphs. 4

- (b) (i) Define an irreducible Markov chain. Prove (or disprove) that the matrix given below is a transition probability matrix of an irreducible Markov chain :
1+5=6

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

- (ii) Prove the additive property of Poisson process. 4

5. Answer **either** (a) **or** (b) :

- (a) (i) The arrivals at a counter in a bank occur in accordance with a Poisson process at an average arrival rate of 8/hour. The duration of service of a customer follows exponential distribution with a mean rate of 6 minutes. Find the following :
- (i) The probability that an arriving customer has to wait. 2
- (ii) The average number of customers in the queue. 2
- (iii) The average number of customers in the system. 6

- (iv) The probability that there are two customers in the system.
- (v) The average waiting time in the queue.
2+2+2+2+2=10

(b) (i)

Define the following states of Markov chain :
1+1+1+1=4
Absorbing state, Persistent state, Transient state and Ergodic state

- (ii) Given the following transition probability matrix

$$P = \begin{pmatrix} q & p \\ p & q \end{pmatrix} \text{ where } p + q = 1$$

Find the probability of transition from state 0 to state 1 in m steps. 6

6. Answer **either** (a) **or** (b) :

- (a) (i) Differentiate between steady state and transient state of a queuing system. 2
- (ii) Define a stationary process. 2
- (iii) Write an explanatory note on basic characteristics of a queuing system. 6

(b) (i) Write a detailed note on applications of stochastic processes. 5

(ii) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that

$$P\{N(s) = k / N(t) = n\} = \binom{n}{k} \left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}$$

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