Total number of printed pages–7

44 (1) CIT0100304

2023

MATHEMATICS–I

Paper : CIT 0100304

Full Marks : 60

Time : $2^1/_2$ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 8 = 8$

(a) State the principle of mathematical induction.

(b) Give an example of a transitive relation.

(c) State the principle of inclusion and exclusion for two sets.

(d) State the pigeonhole principle.

(e) Define the terms: Symmetric matrix and Skew-symmetric matrix.

Contd.

(f) Give an example of an invertible matrix.

(g) What do you mean by measure of central tendency ?

(h) If $A = \{1, 2\}$ and $B = \{0, 1, -1\}$, what is the cardinality of $(A \times B)$?

2. Answer ***any six*** of the following questions : $2\times6=12$

(a) Find the mean, median and mode for the following data :

2, 1, 3, 1, 5, 2, 1, 1, 3, 4

(b) For the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 9 & 5 \end{bmatrix}$

find the following :

(i) Rank of A

(ii) Transpose of A

(iii) Determinant of A

(iv) Eigenvalues of A

(c) Express $A = \begin{bmatrix} 1 & 0 \\ 5 & 3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

(d) Give an example of an equivalence relation.

(e) Find the determinant of the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 6 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

(f) A bag contains 4 balls. Two balls are drawn at random without replacement and are found to be blue. What is the probability that all balls in the bag are blue ?

(g) State Bayes' theorem.

(h) Find the probability distribution for the number of doublets in the three throws of a pair of dice.

(i) Ten numbered cards are there from 1 to 15 and two cards are chosen at random such that the sum of the numbers on both the cards is even. Find the probability that the chosen cards are odd-numbered.

(j) If $P(A) = 0.15$, then what is the probability of not A i.e., $P(\text{not } A)$?

3. Answer ***any four*** of the following questions : 5×4=20

(a) Show that the relation R in the set of integers Z given by $R = \{(a, b) \mid |a - b| \text{ is even}\}$ is an equivalence relation.

(b) Show that the function $f: Z \to Z$ defined by $f(x) = 2x$ is one-one but not onto.

(c) Prove using the principle of mathematical induction :

$$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

(d) Prove that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix.

(e) State the Cayley-Hamilton theorem and verify for the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$.

(f) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 \\ 5 & 3 \end{bmatrix}$$

(g) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 6 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

(h) Show that intersection of two equivalence relation is an equivalence relation.

4. Answer ***any two*** of the following questions : 10×2=20

(a) Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary operations.

(i) Ten numbered cards are there from 1 to 15 and two cards are chosen at random such that the sum of the numbers on both the cards is even. Find the probability that the chosen cards are odd-numbered.

(j) If $P(A) = 0.15$, then what is the probability of not A i.e., $P(\text{not } A)$?

3. Answer ***any four*** of the following questions : 5×4=20

(a) Show that the relation R in the set of integers Z given by $R = \{(a, b) \mid |a - b| \text{ is even}\}$ is an equivalence relation.

(b) Show that the function $f : Z \to Z$ defined by $f(x) = 2x$ is one-one but not onto.

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$$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

(d) Prove that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix.

(e) State the Cayley-Hamilton theorem and verify for the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$.

(f) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 \\ 5 & 3 \end{bmatrix}$$

(g) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 6 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

(h) Show that intersection of two equivalence relation is an equivalence relation.

4. Answer ***any two*** of the following questions : 10×2=20

(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$ using elementary operations.

(b) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

(c) Solve the following system of equations using Gauss elimination method :

$$x+y+z=2$$
$$x+2y+3z=5$$
$$2x+3y+4z=11$$

(d) Find the solution of the given 3×3 system using Cramer's rule :

$$x+y-z=6$$
$$3x-2y+z=-5$$
$$x+3y-2z=14$$

(e) Prepare a frequency distribution table for the scores given :

42, 22, 55, 18, 50, 10, 33, 29, 17, 29, 29, 27, 34, 15, 40, 42, 40, 41, 35, 27, 44, 31, 38, 19, 54, 55, 38, 19, 20, 30, 42, 59, 15, 19, 27, 23, 40, 32, 28, 51.

Take the class intervals as 10–20, 20–30, 30–40, 40–50, 50–60.

From the frequency distribution table answer the following questions :

(i) What does the frequency corresponding to the class interval 20–30 indicate ?

(ii) In which class intervals are the scores 10, 20 and 30 included ?

(iii) Find the range of the scores.