2014

ECONOMICS

(Major)

Paper : 3.1

(Elementary Mathematics for Economics)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Write the subsets of the set $A = \{2, 3, 5\}$.

(b) Define a homogeneous function.

(c) When two sets are called disjoint sets?

(d) State when two matrices A and B are conformable for multiplication.

(e) State whether the following statement of equality is an equation or identity, and justify your answer :

$$(x+y)^2 = x^2 + 2xy + y^2$$

(f) Define a diagonal matrix.

(g) Find the limit of the function

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

(h) State the quotient rule of differentiation.

(i) If

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

write the cofactor of the element a_{12}.

(j) Find $\int \frac{1}{x^5} dx$.

2. Answer the following questions : 2×5=10

(a) If

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

show that $(A')' = A$.

(b) State whether the following sets are equal or equivalent :

$A = \{1, 2, 3, 4\}$ and $B = \{3, 2, 1, 4\}$

(c) Can you add

$$A = \begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 9 \end{bmatrix} ?$$

Justify your answer.

(d) Examine the continuity of the function at the point $x = 1$

$$\begin{aligned} f(x) &= x^2 - 2x + 3 \text{ , when } x < 1 \\ &= 1 \text{ , when } x = 1 \\ &= 2x^2 - 3x + 5 \text{ , when } x > 1 \end{aligned}$$

(e) Can you find determinant of a rectangular matrix? Justify your answer.

3. Answer any *four* of the following : 5×4=20

(a) If

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

show that $A^2 - 3I = 2A$, where I denotes identity matrix.

(b) Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

(c) If

$$y = \frac{x^4 + 1}{x^2 + 1}$$

find $\frac{dy}{dx}$.

(d) Given

$$y = \frac{(2x_1 - x_2^2)}{(x_1^2 + 3x_2)}$$

Find $\frac{\partial y}{\partial x_1}$ and $\frac{\partial y}{\partial x_2}$.

(e) Given

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 5 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 5 \end{bmatrix}$$

Find AB.

(f) Evaluate

$$\int_1^2 (x^2 - 2x + 10)\,dx$$

4. Answer any *four* of the following : 10×4=40

(a) Solve the following market model using matrix inversion : 10

$$Q_d = 50 - 2P$$
$$Q_s = -10 + 3P$$
$$Q_d = Q_s$$

(b) Find $\frac{dy}{dx}$: 5+5=10

(i) $y = \log x(10 + e^x)$

(ii) $y = \frac{1 - vx}{1 + \sqrt{x}}$

(c) Find : 5+5=10

(i) $\int (4x - 5)^6\,dx$

(ii) $\int \frac{1}{x \log x}\,dx$

(d) (i) Give the geometrical interpretation of

$$\int_a^b f(x)\,dx$$

(ii) Given the marginal cost function

$$MC = 3Q^2 - 12Q + 18$$

where Q is output. Find the total cost (TC) function. 5+5=10

(e) (i) Derive the total revenue function $R(Q)$, given the marginal revenue function as $R'(Q) = 100 - 0 \cdot 5Q$. 4

(ii) If

$$A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & -1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 2 & 3 \end{bmatrix}$$

find a matrix C of order 2×4 such that $A - C = 3B$. 6

(f) In a three-sector economy, the input coefficient matrix and final demand vector are as given below :

$$A = \begin{bmatrix} 0 \cdot 3 & 0 \cdot 2 & 0 \cdot 3 \\ 0 \cdot 1 & 0 \cdot 3 & 0 \cdot 4 \\ 0 \cdot 2 & 0 \cdot 3 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 500 \\ 700 \\ 600 \end{bmatrix}$$

Find the sectoral output X_1, X_2 and X_3 using Cramer's rule. 10

(g) (i) If

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{and } C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

prove that $(AB)C = A(BC)$. 8

(ii) Prove that if one row (or column) of a determinant is a multiple of any row (or column), the value of the determinant will be zero. 2

(h) Write short notes on the following : 5+5=10

(i) Partial differentiation and Total differentiation

(ii) Assumptions of static input-output model

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